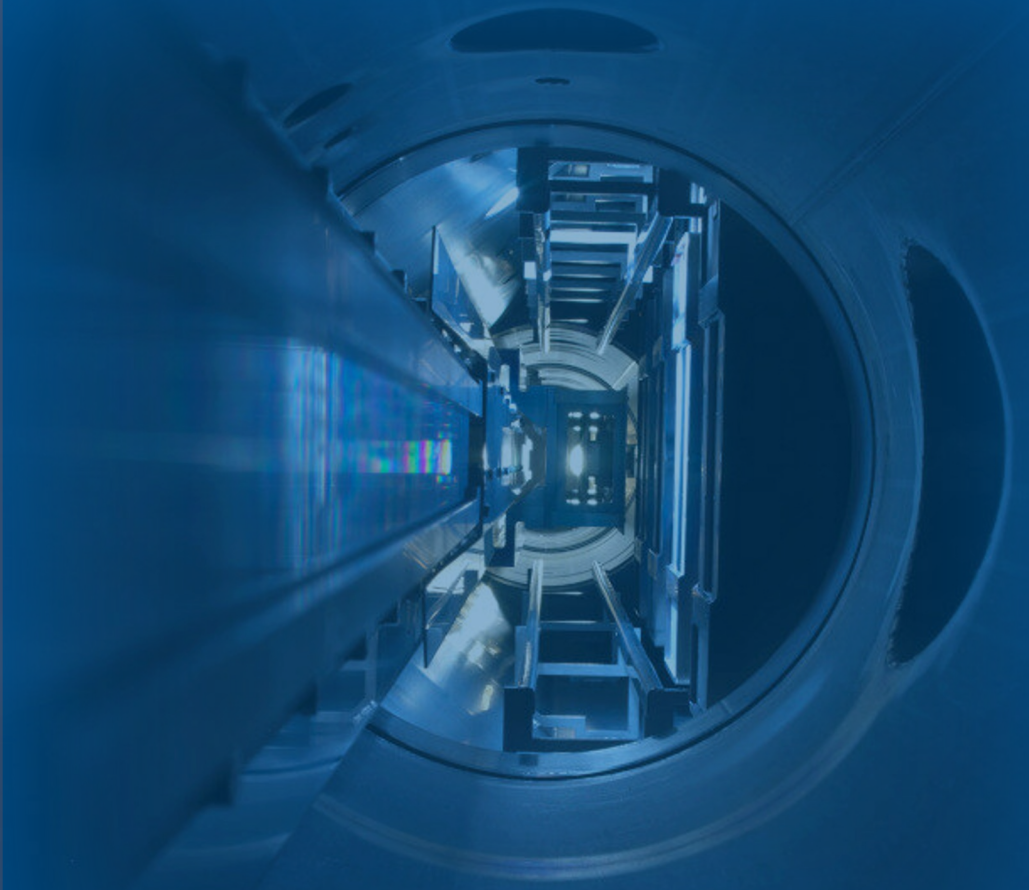
SUPERFLAT



Error estimation for LEEP measurements

Co-funded by the European Union

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This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement N° 101004728

Disclaimer

This is work in progress.

We aim at providing the actual uncertainty bars (incl. systematic error), but we do not have a solution: Some conjectures are made, some steps are unsolved.

Often solutions mean traceability to a standard.

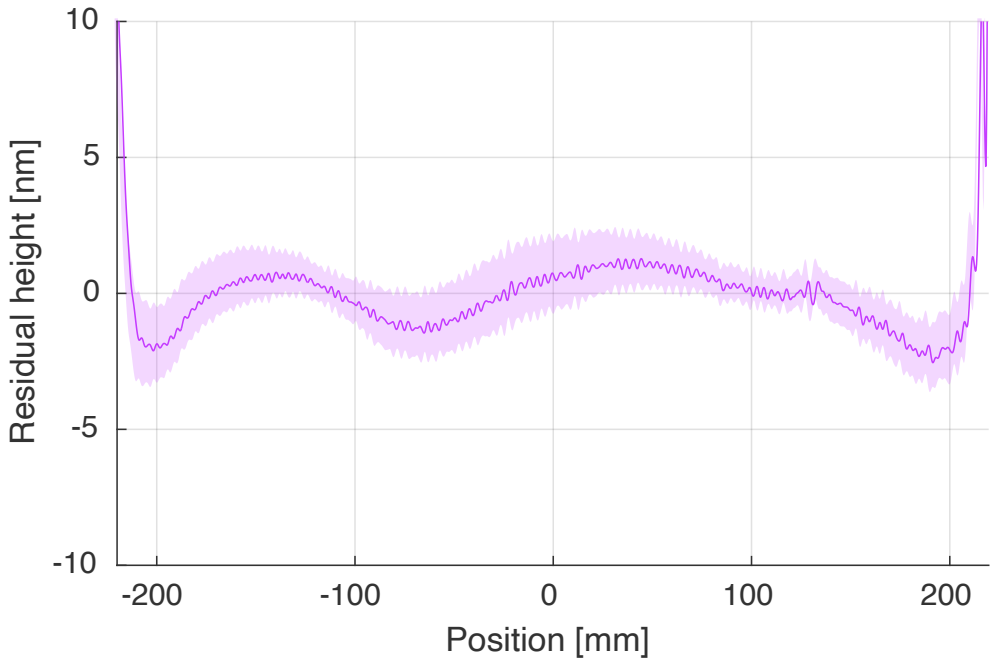
We present ONE posible approach to the problem, not the only one.

All slides but one show LEEP for NOM results, but they same concepts apply to LEEP for stitching interferometry.

Discussion is welcome.

Can we have error bars on our measured surface profiles?

*Can we **guarantee** that the profile of this mirror is within the shadowed area? (P≥99.5%)*



- How confident are we that another instrument (different in nature) would obtain the same result?

- How confident are we that another very good lab would obtain the same result?

- How confident are we that we would obtain the same profile 6 months later.

- How confident are we that the mirror will perform well at the beamline?

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Random Errors

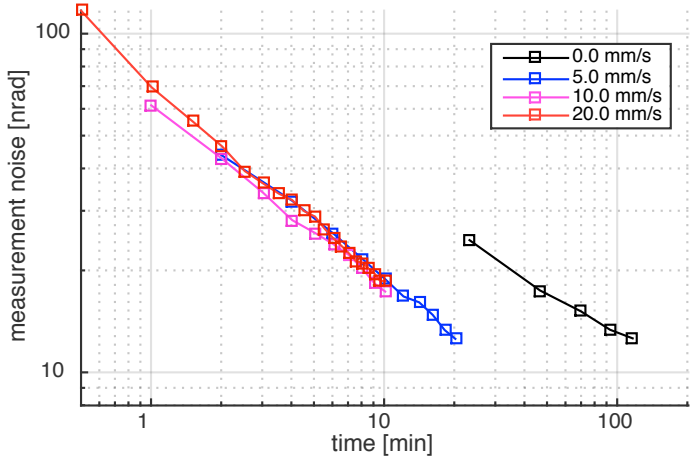
All of us have worked hard to reduce the random errors of our instruments.

Fast random processes:

- Air turbulence, mechanical reproducibility errors, electronic noise of sensors, Acoustic Noise, vibrations.
- Adds **”noise”** to the measured data.
- Normally reduced by averaging (assuming errors are zero mean +additive + uncorrelated)

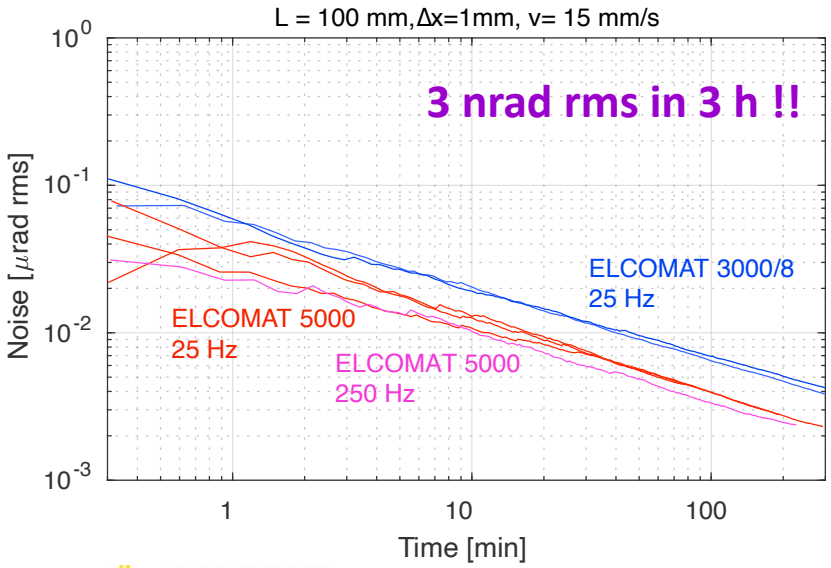
Slow errors:

- Slow temperature drifts, changes of atmospheric pressure, humidity, CO2 contents, etc.
- Measurements work fine except that we have a changing “calibration” of our instruments.
- Typically a change in the **drift** of the measured **curvature** (sometimes a defocus of the sensor).

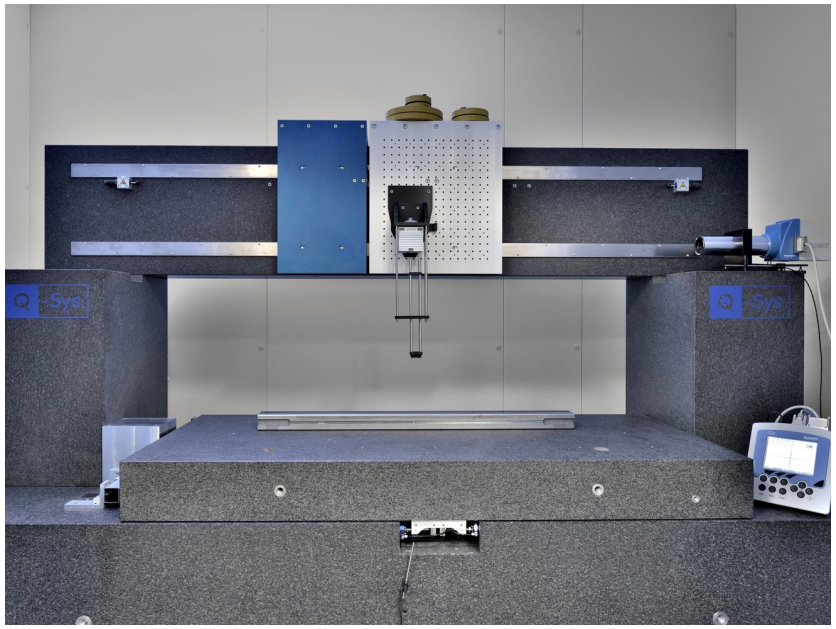


Better Sensors ?
E5k vs E3k
→

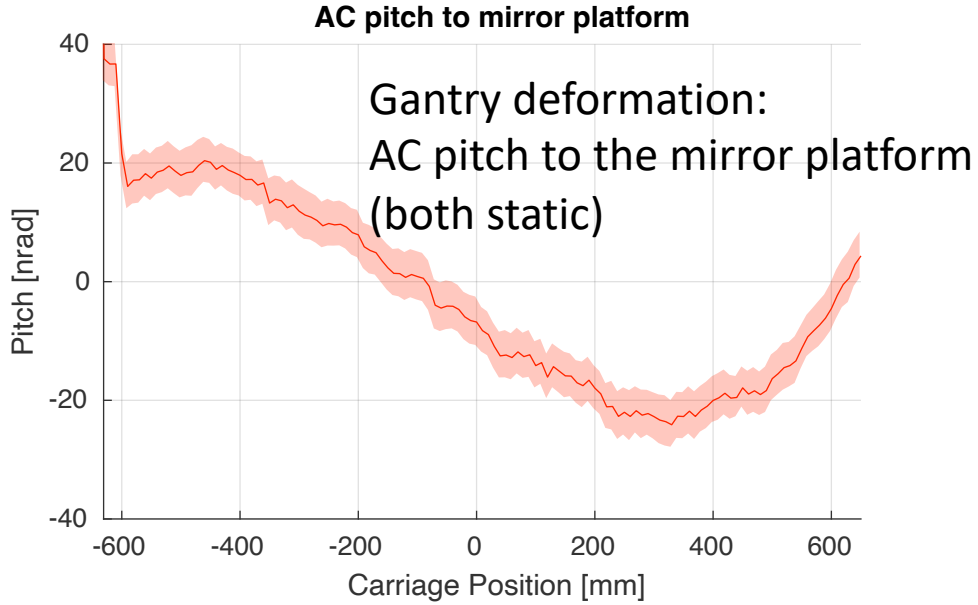
← *Continuous scan: measure fast to measure many times*

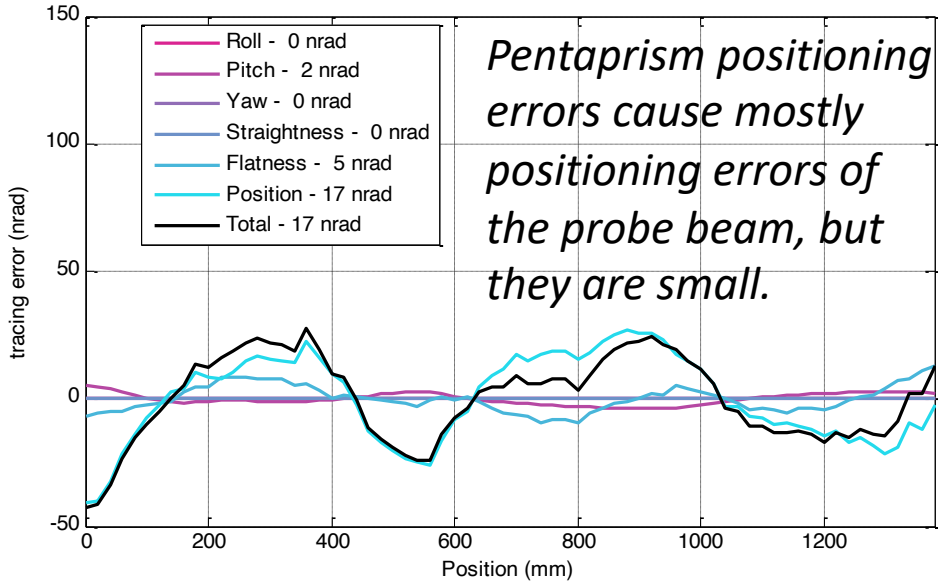


Systematic Errors (NOM)



Autocollimator errors	**700 nrad in 3 mrad, 100 nrad in 0.1 mrad**
Misalignments of AC+PP	ΔR/R ~ 0.01%
Guidance error of PP	20 nrad rms for R=100m L = 1m
Mirror holder reproducibility	?
Temperature gradients (air stratification)	50 nrad for L=1m and ∇T = 0.1K/m
Gantry deformations as we scan	Ca. 40 nrad in L=1.2m (a curvature)
Environment variations	Expansion of encoder scale, refr index of air, expansion of the AC mechanics, etc.





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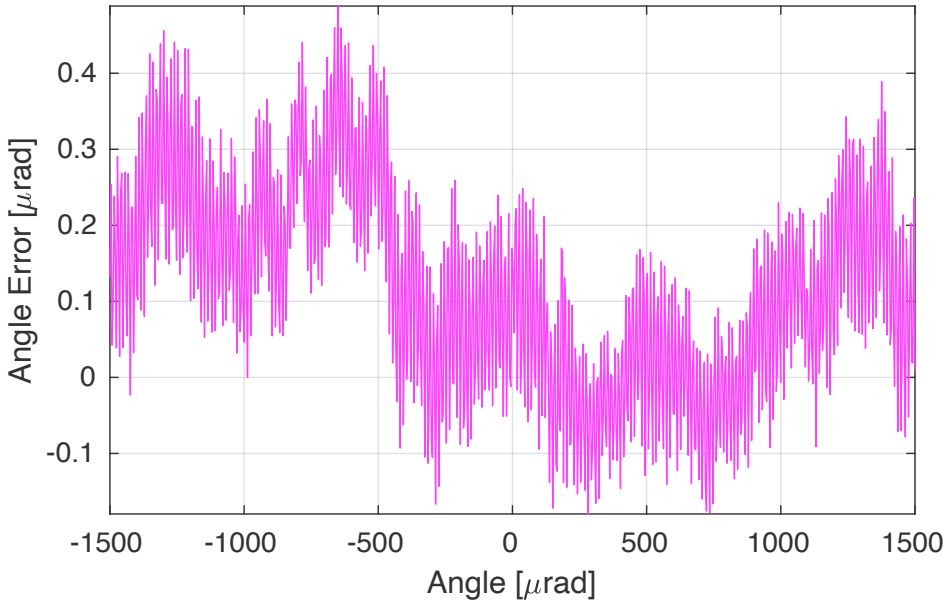
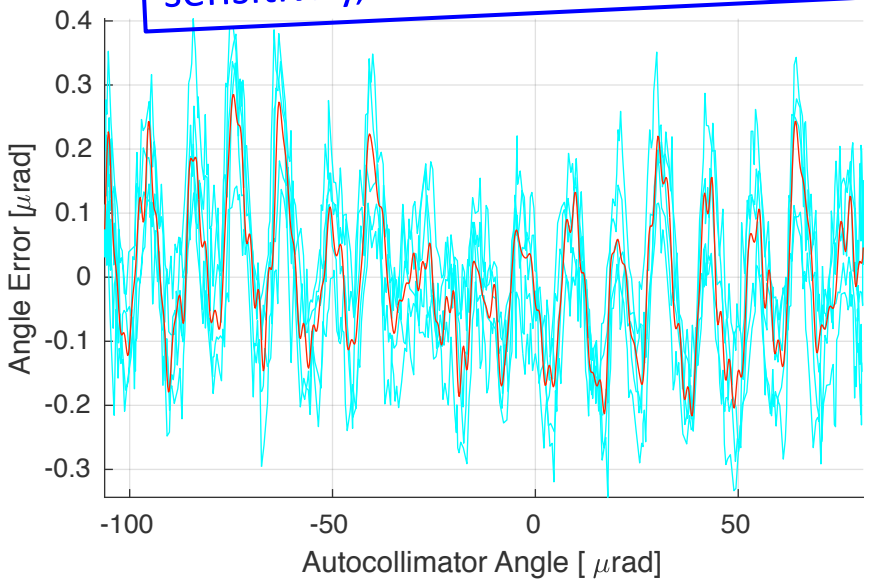
The Calibration of the Autocollimator

Among the sources of systematic error for the NOM the largest contribution is the response of the autocollimator.
- Pixel interpolation error
- Internal reflections ….
- Aberrations are well corrected in the central range.

IMPORTANT NOTE Elcomats are still wonderful instruments, with great long term stability, low heat dissipation, high sensitivity, etc.



ALBA's E5k Calibration in 3 mrad, **700 nrad** rms
Pixel interpolation period = 13.27 µrad
Pixel interpolation amplitude ~100 µrad



ALBA's E3k/8 Calibration against interferometer, showing 150 nrad amplitude errors.

Geckeler et al, NIMA 616 (2010)

Linearity Error Elimination Procedure

Polack et al, NIMA 616 (2010)

Construct the following ***Error Model***

$$M(x,p) = s(x) + T_p + L(M(x,p))$$

Angle measured at position x In measurement p

"True" slope of the mirror at position x

Average pitch depends only on p

Instrument Error

The model assumes that measurements have some error with the following characteristics.

- *The error is* ***additive****.*
- *The error depends only on the angle (It is* ***independent of the position****)*
- *The error* ***does not drift*** *(it is the same for all measurements)*

We need to perform several measurements with ***different pitch angles*** *to have a solvable system of equations.*

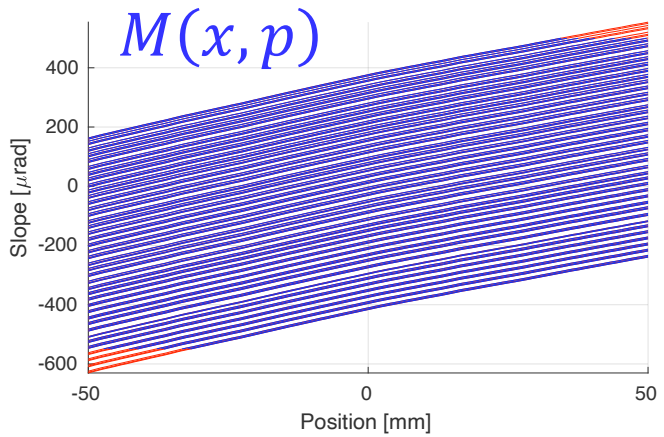
Solutions

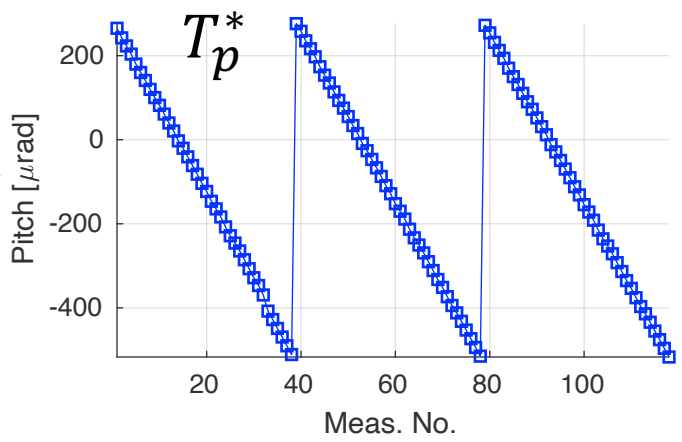
The software solves a large system of equations, overdetermined, as a least square optimization.

It returns a solution that provides the ***best fit to the measurement data according to the model***

$$M(x,p) = s(x) + T_p + L(M(x,p))$$

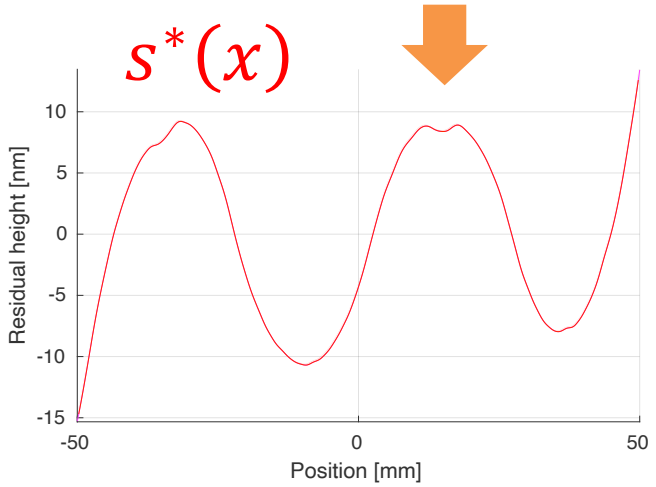
Measured data

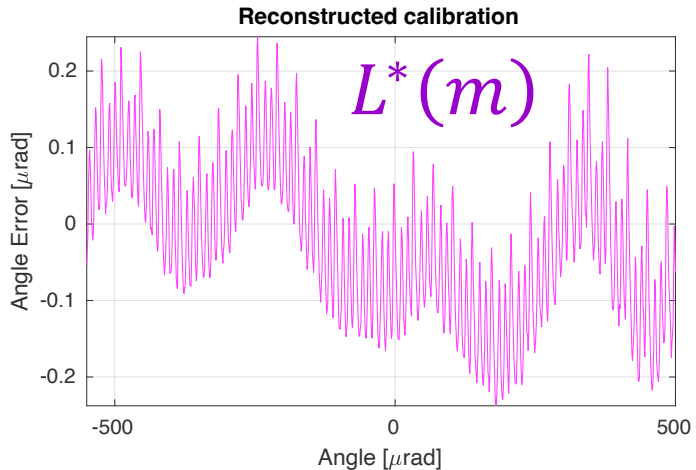




Reconstructed pitch

Reconstructed Surface (height error, mean ellipse detrended)





Reconstructed error

Check how good is the best fit (fit residue)

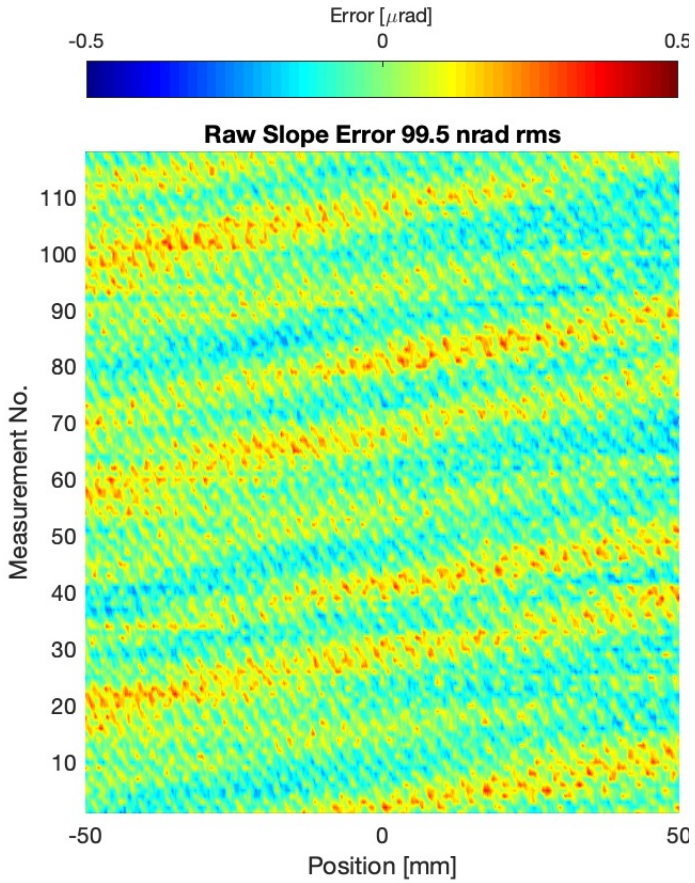
We can feed back the estimated functions to our model expresión, to obtain the residue of the fit.

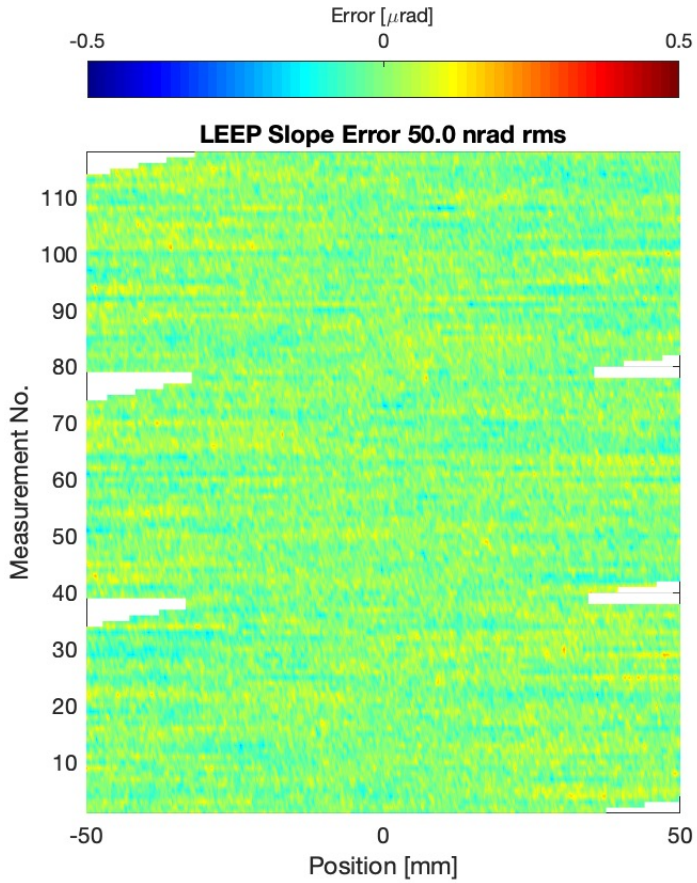
$$\varepsilon(x,p) = M(x,p) - M^*(x,p)$$

$$M^*(x,p) = s^*(x) + T_p^* + L^*(M(x,p))$$

$M(x,p) = s(x) + T_p$

$M(x,p) = s(x) + T_p + L(M(x,p))$





***RAW** model, asumes the AC measures the true angle.*

There are clear correlations

Some error term is missing

LEEP model,

Only decorrelated noise is left.

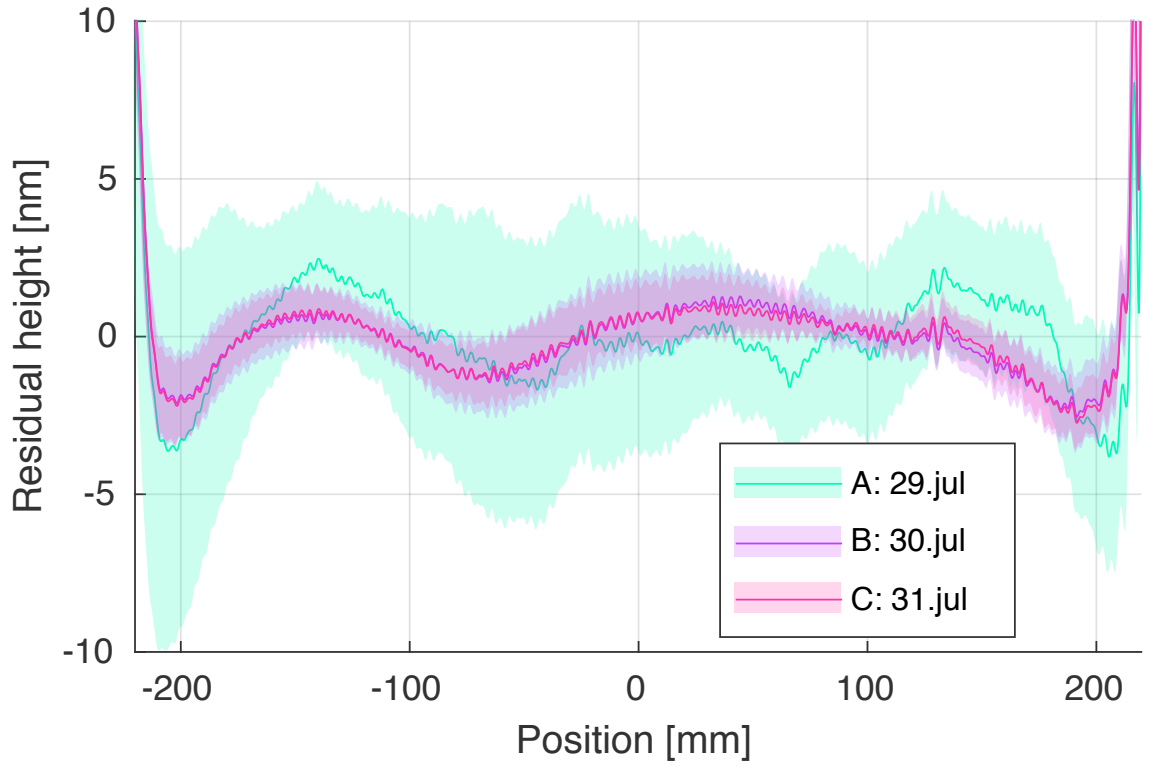
The solution fits well the data

ERFLAT

Error bars

We now have N_p measurements for which the remaining error is just ***uncorrelated noise (random)***
So our error bars could be $\sigma/\sqrt{N_p}$

$$Err^2(x) = \frac{1}{N_p}\langle|\varepsilon(x,p)|^2\rangle_p$$



Measurement on one mirror as it stabilizes after installing it at the NOM.

The lines of the different measurerments do not really match, but their "error bands" do.

(Nevertheless, errors here are dominated by noise)

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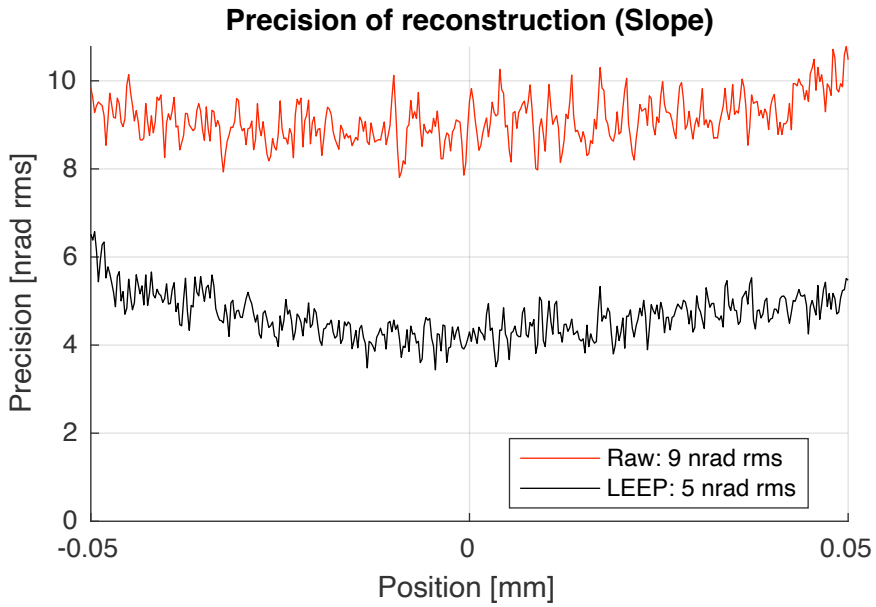
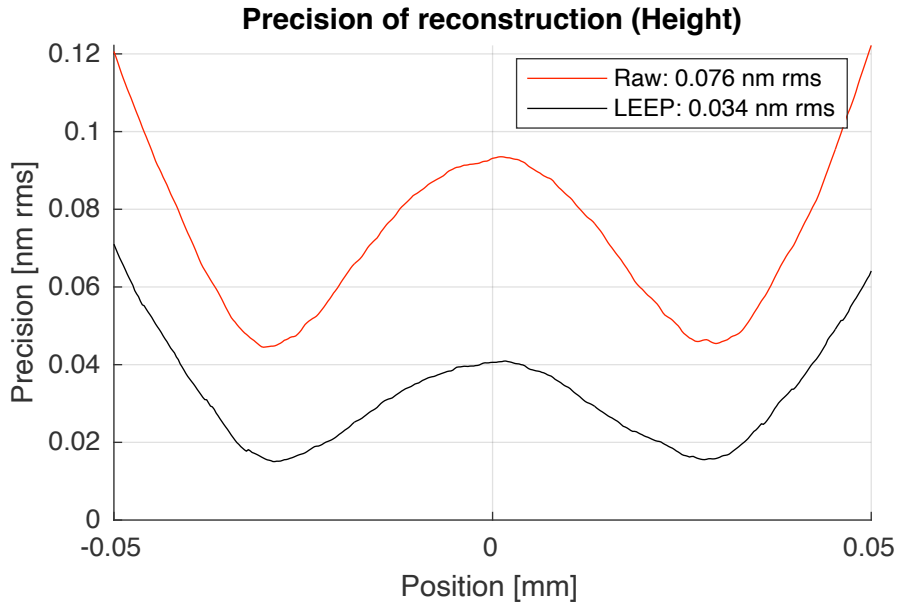
Picometer accuracy, really?

*Some actual results (MoonPics ellipse) show "accuracy" errors as small as **5 nrad** or **34 pm** (rms)*
Depending on length, curvature and time available for measurements it is usual to get errors about 150 pm





Moreover the system warns you if your data does not fit the model:

In this case the W shape in the error bar comes from drifts on the measured curvature.
"Easy" to fix:

$$\begin{cases} M(x,p) = s(x) + T_p + L(M(x,p)) + \kappa_p x^2 \\ 0 = \langle s''(x) \rangle_x \end{cases}$$

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But we know that NOM has other error contributions LEEP does not consider.

Why don't they show up in the fit residue?

Ambiguities

The model has ambiguities: ***Different surfaces*** *+ errors that produce the* ***same measurement dataset*** *(and therefore they all fit the model equally well)*

1) Additive errors that depend only on position are indistinguishable from the slope profile

$$s(x) \rightarrow \eta s(x) + (1-\eta) r(x)$$

Any value of η produces the same measured data

This is the case of pentaprism positioning errors, Alignment errors, gantry deformations, air stratification…. (About 20 to 50 nrad R_err ~ 20,000 km)

2) Global scaling of the slope is not captured (solvable with independent pitch measurement)

$$s(x) \rightarrow \frac{s(x)}{1+\eta} \qquad T_k \rightarrow \frac{T_k}{1+\eta} \qquad L(m) \rightarrow \eta m + (1-\eta) L(m)$$

unfortunately η depends on temperatura, humidity, and is a drifting parameter.

3) Errors that depend on local curvature but not on angle.

Frustratingly experienced in toroidal mirrors.
Curvature defocuses the reticle, and this depends on the working distance.
Some cases reduce to case 1)

… are there other ambiguities ?

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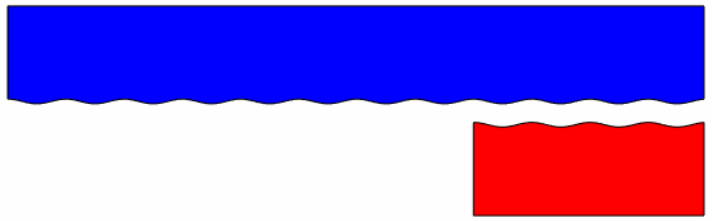
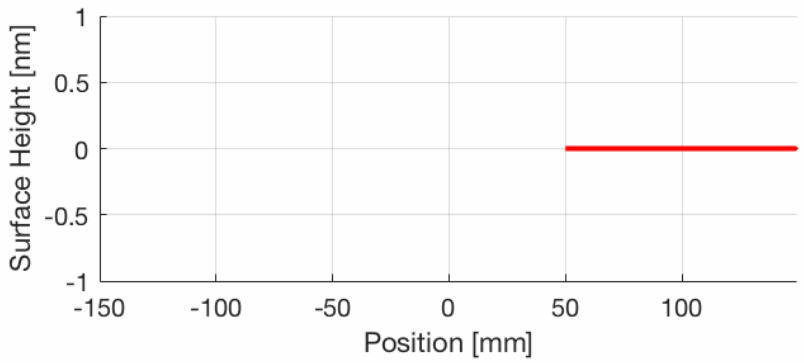
Ambiguities for stitching interferometry

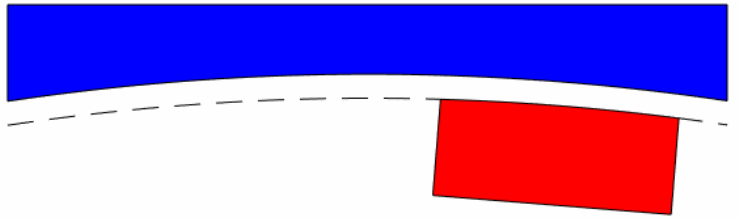
For stitching interferometry we know the ambiguities (Periodic errors and curvatures)
We have already sophisticated the model to "remove" them (in 1D and 2D)
And there is proof that there are no other ambiguities?

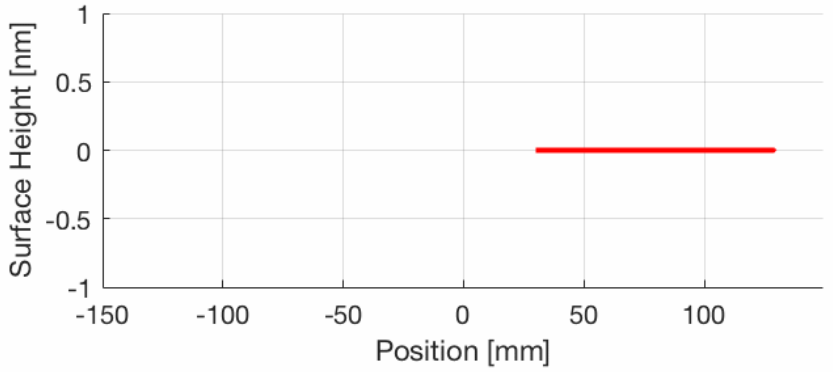
Periodic errors





Curvatures, twist





[1] *Nicolas et al IWXM'18*
[2] *https://doi.org/10.1364/OE.26.027212*

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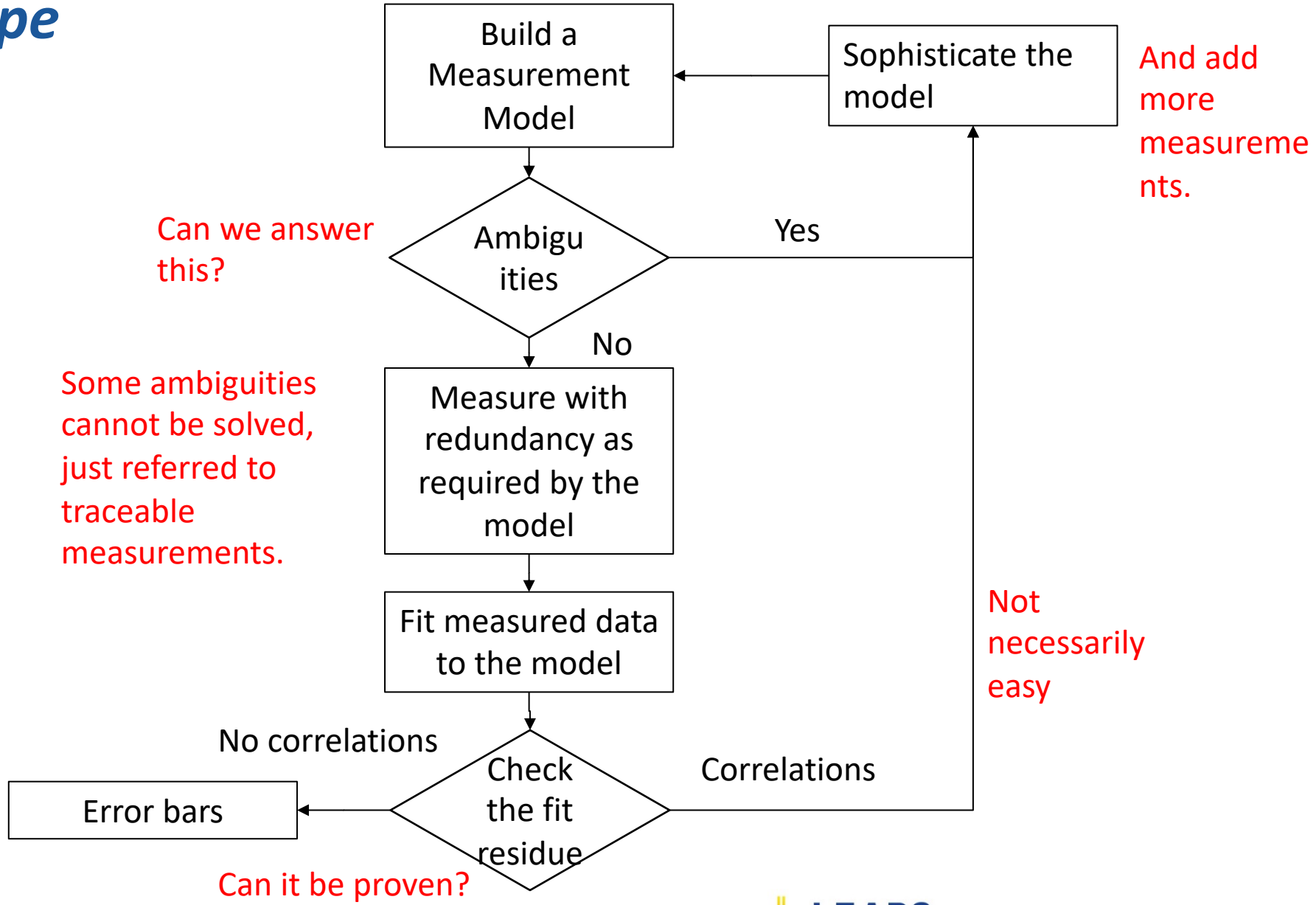
Almost there…

Is it possible to have some type of error that:
*a)is **NOT AN AMBIGUITY** of the model, and*
*b)gives **NO CORRELATIONS** in the fitting residue?*

I would say it is NOT but we have no proof.

The recipe



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Open questions

Should we **record environment conditions** (Temperature, Humidity, Pressure) in our metrology data?

Should we all measure in identical conditions?

In any case, how do we use environment parameters to correct the profiles, or to explain the differences?

Simulation: should we develop (and share) simulation software to account for known error sources, for stitching interferometry and scanning deflectometry.

Can we proof we have accounted for all **ambiguities**?

Do all non-ambiguity errors produce correlations in the fit residue?

Thank you